Inverse of Exponentials and More Log Properties (2.10 + 2.12) Homework

1. Let $f(x) = \log_4(x)$. A horizontal dilation of this function by a factor of $\frac{1}{2}$ is equivalent to a vertical translation by k units. Find k.

2. Let $g(x) = \log_3(x)$. A vertical translation -2 units is equivalent to a horizontal dilation by a factor of c. Find c.

3. Let $f(x) = x^3$ and let $g(x) = \ln(x)$. The graph of $(g \circ f)(x)$ is the image of the graph of $g(x)$ after a...
   (A) Vertical dilation by a factor of 3
   (B) Vertical dilation by a factor of $\frac{1}{3}$
   (C) Horizontal dilation by a factor of 3
   (D) Horizontal dilation by a factor of $\frac{1}{3}$

4. Three of the following functions are equivalent. One is not. Which one of the following is not equivalent to the others.

   $f(x) = \dfrac{1}{3}\log_2 x$ $\qquad$ $g(x) = \log_2\left(\sqrt[3]{x}\right)$ $\qquad$ $h(x) = \log_8 x$ $\qquad$ $k(x) = \log_9 \sqrt{x}$

5. Let $f(x) = \log_9 x$ and let $g(x) = \log_3 x$. The graph of $f(x)$ is the image of the graph of $g(x)$ after a vertical dilation by a factor of k. Find k.

6. Let $f(x) = 5^x$. Find $f^{-1}\left(\frac{1}{25}\right)$.

7. Let $g(x) = \left(\frac{1}{9}\right)^x$. Find $g^{-1}(27)$.

8. Match each table with the model type that most appropriately matches the points. Use each table exactly once and each function type exactly once.

   $f(x) = ax + b$ $\qquad$ $g(x) = a(b)^x$ $\qquad$ $h(x) = a\log_b x$ $\qquad$ $k(x) = ax^2 + bx$

Table 1

x	y
-2	4.5
0	9
2	18
4	36
6	72

Table 2

x	y
3	6
6	12
9	18
12	24
15	30

Table 3

x	y
6	10
18	15
54	20
162	25
486	30

Table 4

x	y
-4	14
0	5
4	0
8	-1
12	2

9. Algebraically prove that $f(x) = 5^x$ and $g(x) = \log_5 x$ are inverse functions.

10. Simplify each of the following. Each simplifies to an integer value.
   a. $5^{2\log_5 4}$
   b. $\frac{\log_5 8}{\log_5 2}$
   c. $\log_6(2^5) + \log_6(3^5)$
   d. $3^{\log_3 4 + \log_3 2}$
   e. $\log_6 2 + \log_6 18$
   f. $\log_3(3^{10}) - \log_3(3^6)$

11. (DESMOS/CALCULATOR)
Saisha hears about a new type of cryptocurrency called "Lightning Coin" and begins mining it. Six days after learning about lightning coins ($t = 6$), she has a total of 17 lightning coins. Thirteen days after learning about them ($t = 13$), she has a total of 28 lightning coins.

The total number of lightning coins Saisha has can be modeled by $N(t) = a\ln(t+2) + b$, where $N(t)$ is the total number of coins and t is days since she first learned about lightning coins.

A.
   i. Use the given data to write a system of equations that can be used to find the values for a and b.
   ii. Find the values for a and b.

B.
   i. Use the given data to find the average rate of change of the total number of lightning coins Saisha has from $t = 6$ to $t = 13$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
   ii. Use the average rate of change found in B (i) to estimate the total number of lightning coins Saisha has for $t = 18$ days. Show the work that leads to your answer.
   iii. Let A_t represent the estimate of the total number of lightning coins Saisha has using the average rate of change found in part B (i). For A_{18} found in part B (ii), it can be shown that $A_{18} > N(20)$. Explain why, in general, $A_t > N(t)$ for all $t > 13$. Your explanation should include a reference to the graph of N and its relationship to A_t.

C. The logarithmic function N has exactly one zero. That zero can be used to determine a domain restriction for N. Based on the context of the problem, explain how that zero can be used to determine a boundary for the domain of N.

12. Rewrite as a single logarithm without negative exponents in any part of the expression. Your result should be of the form $\log$(expression):
$$\frac{1}{3}\log(x) - \frac{4}{3}\log(x) - \log(4)$$

13. Rewrite as a single logarithm base 5 without negative exponents in any part of the expression. Your result should be of the form $\log_5$(expression):
$$3\log_5(x+1) - 2\log_5(x-2) - \log_5 10$$

14. Rewrite as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be of the form $\log_2$(expression):
$$\log_2(x) - \log_8(x) - 2\log_2(2x-1)$$

15. Rewrite as a single natural logarithm without negative exponents in any part of the expression. Your result should be of the form $\ln$(expression):
$$3\ln(2x) - 2\ln(3x^2)$$

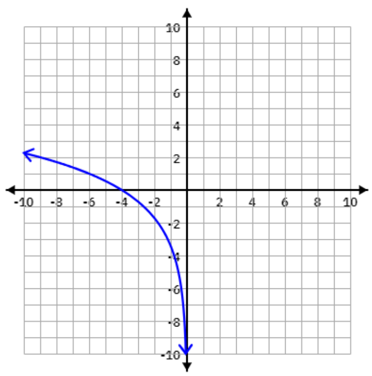
16. The exponential function $f(x) = a^x$ contains the points $(3,4)$ and $(9,64)$. Find the average rate of change of $g(x) = \log_a x$ over the interval $[4,64]$.

17. Let $f(x) = a^x$. If $f(x)$ decreases at an increasing rate, what can be said about $g(x) = \log_a x$ over its domain?
    (A) $g(x)$ increases at a decreasing rate
    (B) $g(x)$ decreases at a decreasing rate
    (C) $g(x)$ increases at an increasing rate
    (D) $g(x)$ decreases at an increasing rate

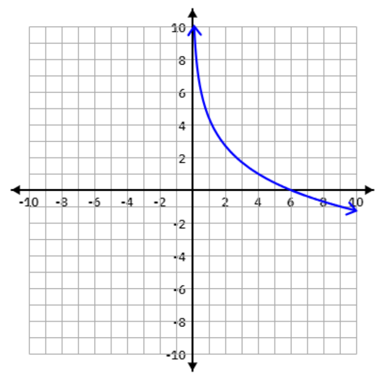
18. Below are four functions: f, g, h, and k. Below those are eight graphs. Four are the graphs of f, g, h, and k. The other four are f^{-1}, g^{-1}, h^{-1}, and k^{-1}. Identify which graph represents which function (or inverse function).

$f(x) = -4\left(\frac{3}{2}\right)^x$ $\qquad$ $g(x) = 6\left(\frac{2}{3}\right)^x$ $\qquad$ $h(x) = 3(2)^{x-1}$ $\qquad$ $k(x) = -9\left(\frac{1}{3}\right)^x$

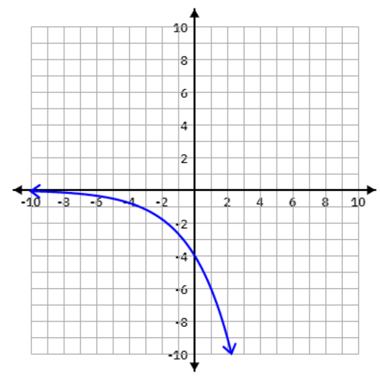
(A)



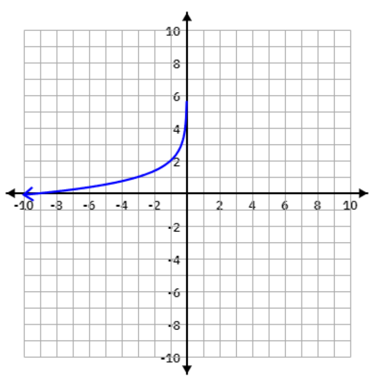
(B)



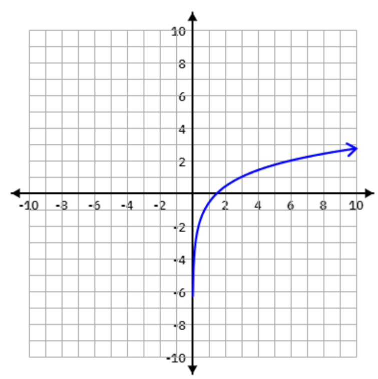
(C)



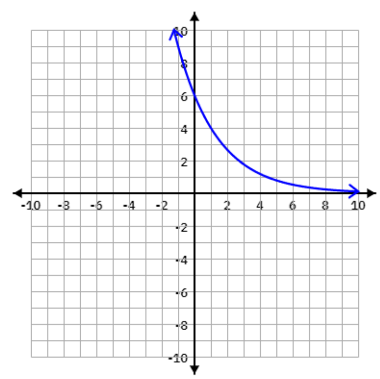
(D)



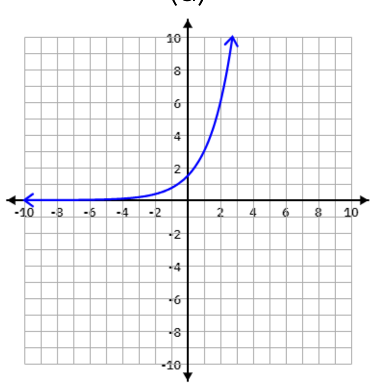
(E)



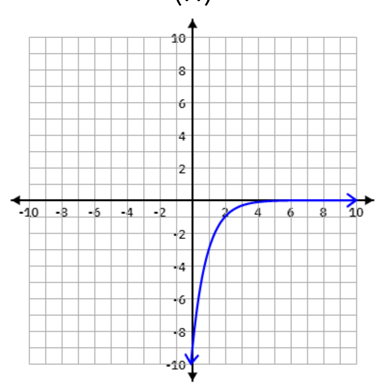
(F)



(G)

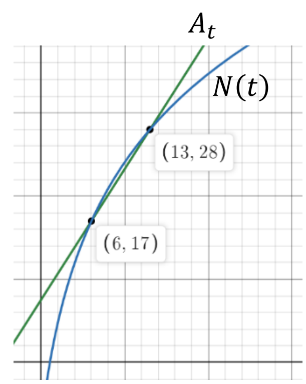


(H)



Inverse of Exponentials and More Log Properties HW Answers

1. $k = \frac{1}{2}$
2. $c = 9$
3. (A) Vertical dilation by a factor of 3
4. $k(x) = \log_9 \sqrt{x}$
5. $k = \frac{1}{2}$
6. -2
7. $-\frac{3}{2}$
8. Table 1: $g(x)$ (exponential)
   Table 2: $f(x)$ (linear)
   Table 3: $h(x)$ (logarithmic)
   Table 4: $k(x)$ (quadratic)
9. $(f \circ g)(x) = f(g(x)) = f(\log_5 x) = 5^{\log_5 x} = x$
   $(g \circ f)(x) = g(f(x)) = \log_5(5^x) = x$
   Since $(f \circ g)(x) = (g \circ f)(x) = x$, f and g are inverse functions.
10.
    a. 16
    b. 3
    c. 5
    d. 8
    e. 2
    f. 4
11.
    A.
        i. $17 = a\ln(6+2) + b$
           $28 = a\ln(13+2) + b$
        ii. $a = 17.499$ and $b = -19.388$
    B.
        i. $\frac{N(13)-N(6)}{13-6} = \frac{28-17}{13-6} = \frac{11}{7} = 1.571$
        ii. $\frac{N(18)-17}{18-6} \approx \frac{11}{7}$
        $$N(18) - 17 \approx 18.85714$$
        $$N(18) \approx 35.857$$
        iii. We are comparing the outputs of the concave down function $N(t)$ to outputs of the secant line A_t that passes through N at $t = 6$ and $t = 13$. Because N is concave down, the graph of the secant line is above the graph of N after $t = 13$. So, for $t > 13$, $A_t > N(t)$.



    C. $N(t) = 0 \rightarrow t = 1.028$
       For $t < 1.028$, $N(t)$ has a negative output.
       But Saisha cannot own a negative number of lightning coins.
       So, the domain of $N(t)$ should be restricted to $t \geq 1.028$.
12. $\log\left(\frac{1}{4x}\right)$
13. $\log_5\left(\frac{(x+1)^3}{10(x-2)^2}\right)$
14. $\log_2\left(\frac{x^{\frac{2}{3}}}{(2x-1)^2}\right)$
15. $\ln\left(\frac{8}{9x}\right)$
16. Note that $f(x)$ and $g(x)$ are inverses.
    Since $f(3) = 4$, we know $g(4) = 3$.

Since $f(9) = 64$, we know $g(64) = 9$.

Average rate of change of g over $[4,64]$ is $\frac{g(64)-g(4)}{64-4} = \frac{9-3}{64-4} = \frac{6}{60} = \frac{1}{10}$

17. (D) $g(x)$ decreases at an increasing rate

18.

Graph A: $f^{-1}(x)$ Graph B: $g^{-1}(x)$ Graph C: $f(x)$

Graph D: $k^{-1}(x)$ Graph E: $h^{-1}(x)$ Graph F: $g(x)$

Graph G: $h(x)$ Graph H: $k(x)$